



TECHNICAL INFORMATION

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THE AFFINITY LAWS & SPECIFIC SPEED IN CENTRIFUGAL PUMPS

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AFFINITY LAWS

The performance of a given pump at various speeds and the performance of geometrically similar pumps is governed by a set of formulas known as the *affinity laws*. The affinity laws, or formulas, state that for a given pump, the capacity will vary directly as the speed, the head and NPSH will vary as the square of the speed, and the required horsepower will vary as the cube of the speed or, mathematically:

$$Q \propto N \quad (1)$$

$$H \propto N^2 \quad (2)$$

$$NPSH \propto N^2 \quad (3)$$

$$HP \propto N^3 \quad (4)$$

where: Q = pump capacity in gallons, per minute at best efficiency point (b.e.p.)

H = pump head in feet at b.e.p.

NPSH = required NPSH in feet at b.e.p.

HP = required horsepower at b.e.p.

The affinity laws also govern the performance of geometrically similar pumps; that is, pumps which are identical except for size. If the performance of a given model pump is known, the performance of a prototype pump can be predicted. A prototype pump is made from a model by multiplying all dimensions of the model by the same factor. This size factor is denoted as KD. In this case, the prototype pump and the model are said to be *homologous* to each other.

In the case of homologous pumps, the affinity laws state that, if the model and prototype are run at the same speed, the prototype capacity will equal the model capacity times the cube of the size factor, the head and NPSH of the prototype will equal the head and NPSH of the model times the square of the size factor, and the required horsepower of

the prototype will equal the horsepower of the model times the size factor to the fifth power, or:

$$Q_p = KD^3 \times Q_m \quad (5)$$

$$H_p = KD^2 \times H_m \quad (6)$$

$$NPSH_p = KD^2 \times NPSH_m \quad (7)$$

$$HP_p = KD^5 \times HP_m \quad (8)$$

where the subscripts p and m denote prototype and model, respectively.

If the prototype is to run at a different speed than the model, the two foregoing sets of equations can be combined to determine prototype performance:

$$Q_p = KD^3 \times \left(\frac{RPM_p}{RPM_m} \right) \times Q_m \quad (9)$$

$$H_p = KD^2 \times \left(\frac{RPM_p}{RPM_m} \right)^2 \times NPSH_m \quad (10)$$

$$NPSH_p = KD^2 \times \left(\frac{RPM_p}{RPM_m} \right)^2 \times NPSH_m \quad (11)$$

$$HP_p = KD^5 \times \left(\frac{RPM_p}{RPM_m} \right)^3 \times HP_m \quad (12)$$

The following three examples show how these equations are used.

Example 1. Given: a pump with the following performance at best efficiency point (b.e.p.):

$$Q = 1000 \text{ GPM}$$

$$H = 150 \text{ Ft.}$$

$$\text{required NPSH} = 11 \text{ Ft.}$$

$$HP = 45$$

$$\text{Speed} = 1750 \text{ RPM}$$

What will the performance of this pump be at b.e.p. when run at 2900 RPM? Using equations (1), (2), (3) and (4):

$$Q = 1000 \times \left(\frac{2900}{1750}\right) = 1660 \text{ GPM}$$

$$H = 150 \times \left(\frac{2900}{1750}\right)^2 = 411 \text{ Ft.}$$

$$\text{NPSH} = 11 \times \left(\frac{2900}{1750}\right)^2 = 30.2 \text{ Ft.}$$

$$\text{HP} = 45 \times \left(\frac{2900}{1750}\right)^3 = 205 \text{ H.P.}$$

Example 2. Given: the same pump as in Example 1. The impeller diameter is 13". What would be the b.e.p. performance of a homologous pump with an impeller diameter of 22" when run at 1750 RPM?

First, K_D must be calculated:

$$K_D = \frac{22}{13} = 1.69$$

Then, using equations (5), (6), (7) and (8) :

$$Q_p = (1.69)^3 \times 1000 = 4850 \text{ GPM}$$

$$H_p = (1.69)^2 \times 150 = 430 \text{ Ft.}$$

$$\text{NPSH}_p = (1.69)^2 \times 11 = 31.5 \text{ Ft.}$$

$$\text{HP}_p = (1.69)^5 \times 45 = 625 \text{ H.P.}$$

(In actual practice, the required horsepower of the prototype will be somewhat less since larger pumps tend to be more efficient).

Example 3. Given: a pump with the following performance at best efficiency point:

$$Q = 500 \text{ GPM}$$

$$H = 350 \text{ Ft.}$$

$$\text{required NPSH} = 10 \text{ Ft.}$$

$$\text{HP} = 55$$

$$\text{Speed} = 3500 \text{ RPM}$$

$$\text{Impeller Diameter} = 10\frac{1}{2}"$$

What will be the performance of a homologous pump with an impeller diameter of 20 inches when run at 1170 RPM?

First, $K_D = \frac{20}{10.5} = 1.905$

Then, using equations (9), (10), (11) and (12):

$$Q_p = (1.905)^3 \times \left(\frac{1170}{3500}\right) \times 500 = 1156 \text{ GPM}$$

$$H_p = (1.905)^2 \times \left(\frac{1170}{3500}\right)^2 \times 350 = 142 \text{ ft.}$$

$$\text{NPSH}_p = (1.905)^2 \times \left(\frac{1170}{3500}\right)^2 \times 10 = 4.06 \text{ ft.}$$

$$\text{HP}_p = (1.905)^5 \times \left(\frac{1170}{3500}\right)^3 \times 55 = 51.5$$

SPECIFIC SPEED AND SUCTION SPECIFIC SPEED

Both specific speed, N_s , (or more correctly, discharge specific speed), and suction specific speeds, S , are mathematical derivations of the affinity laws. They are defined as:

$$N_s = \frac{\text{RPM} \sqrt{\text{GPM}}}{H^{3/4}} \quad (13)$$

And

$$S = \frac{\text{RPM} \sqrt{\text{GPM}}}{\text{NPSH}^{3/4}} \quad (14)$$

where GPM, H and NPSH are taken at best efficiency point.

The actual definition of specific speed is the RPM at which a pump geometrically similar (or homologous) to the one under consideration, but reduced in size, would run to produce 1 GPM at 1 ft. head. Similarly, suction specific speed is the RPM at which a pump geometrically similar to the one under consideration, but reduced in size, would produce 1 GPM at 1 ft. required NPSH.

These definitions have no practical significance, but the values of these parameters are very useful and convenient in the classification of pumps.

Specific speed is a *type* number, that is, its value is used to determine the type of pump under consideration or the type of pump required for a given set of conditions. Table I shows the types of pumps for various specific speeds. In general, pumps with a low specific speed are designated "low capacity" and those with a high specific speed are termed "high capacity."

SPECIFIC SPEED RANGE	PUMP TYPE
Below 2,000	Process Pumps, Feed Pumps Turbine Pumps Mixed Flow Pumps Axial Flow Pumps
2,000- 5,000	
4,000-10,000	
9,000-15,000	

TABLE I.

Unlike specific speed, suction specific speed is not a type number, but a criterion of a pump's performance with regard to cavitation. Table II shows how pumps are rated according to their suction specific speeds.

SUCTION SPECIFIC SPEED RANGE		RATING
Single Suction Pumps	Double Suction Pumps	
Above 11,000	Above 14,000	Excellent
9,000 – 11,000	11,000 - 14,000	
7,000 - 9,000	9,000 – 11,000	Average
5,000 - 7,000	7,000 – 9,000	
Below 5,000	Below 7,000	Very Poor

TABLE II.

The following points should be understood about specific speed and suction specific speed:

1. Both specific speed and suction specific speed remain constant regardless of what speed a given pump is run.
2. They are the same for homologous pumps.
3. They are inherent in the design of a given pump.
4. There is no relation between the two; that is, either high or low specific speed pumps can have either high or low suction specific speeds, depending upon the design.

Example 4. Given: the model and prototype pumps in Example 3, determine whether they have the same specific speed and suction specific speed and rate them according to type and cavitation performance.

For the model: using equations (13) and (14):

$$N_s = \frac{3500\sqrt{500}}{350^{3/4}} = 965$$

$$S = \frac{3500\sqrt{500}}{10^{3/4}} = 13,900$$

For the prototype:

$$N_s = \frac{1170\sqrt{1156}}{142^{3/4}} = 965$$

$$S = \frac{1170\sqrt{1156}}{4.06^{3/4}} = 13,900$$

(Actually, these numbers had to come out the same if the calculations in Example 3 are correct since both specific speed and suction specific speed are mathematical derivations of the affinity laws).

Either of these pumps could be classified as "low capacity pump with excellent NPSH characteristics."

In many cases, higher specific speed pumps can be used to great advantage for a given operating condition. To meet a given head and capacity, a higher specific speed pump will run at a higher speed; it will therefore be smaller and thus both pump and driver will be less expensive.

Example 5. Given: the following operating conditions:

Capacity – 10,000 GPM

Head - 100 Ft.

Available NPSH - 32 Ft.

Determine the best pump operating speed assuming the use of a single suction pump, an electric motor with 60 cycle current, and operation at best efficiency point.

Using equations (13) and (14):

$$N_s = \frac{RPM \sqrt{10,000}}{100^{3/4}} = RPM \times 3.14$$

$$S = \frac{RPM \sqrt{10,000}}{32^{3/4}} = RPM \times 7.43$$

From these values, the following chart can be prepared:

SPEED – RPM	REQUIRED SPECIFIC SPEED	REQUIRED SUCTION SPECIFIC SPEED
870	2,740	6,460
1160	3,640	8,620
1750	5,500	13,000
3500	11,000	26,000

From this chart, the following observations can be made:

1. 870 RPM operation and 1160 RPM operation are relatively simple from a design standpoint.
2. 3500 RPM operation is out of the question since a suction specific speed of 26,000 is virtually impossible to attain with conventional pump designs. (Suction specific speeds as high as 40,000 have been attained by the use of cavitating inducers).
3. 1750 RPM operation is most desirable; however, a pump with excellent NPSH characteristics is required.

SUMMARY

1. The affinity formulas are very useful in determining pump performance at different speeds and for determining performance of homologous pumps.
2. Homologous pumps are defined as pumps which are geometrically similar but different in size.
3. Specific speed is a type number and is used to classify pumps.
4. Suction specific speed is used as a measure of cavitation performance of a pump.
5. Specific speed and suction specific speed are applied to a pump only at its best efficiency point.
6. Both specific speed and suction specific speed remain constant for a given pump design regardless of what speed the pump is run at or its size. (In actual practice, large prototype pumps made from small models tend to have a slightly higher specific speed than the model since they tend to peak at a higher capacity than predicted by the affinity laws.)
7. There is no relationship between specific speed and suction specific speed, within limits.
8. High specific speed pumps can be used to advantage in many cases since, for a given condition, their operating speed is higher and the pump is therefore smaller and less expensive. If NPSH is critical, the suction specific speed may limit the speed at which the pump can be run.

Therefore, when selecting pumps for a given condition, it is well to consider these relationships. A smaller, less expensive pump can often be the result.

SPECIFIC SPEED NOMOGRAM

