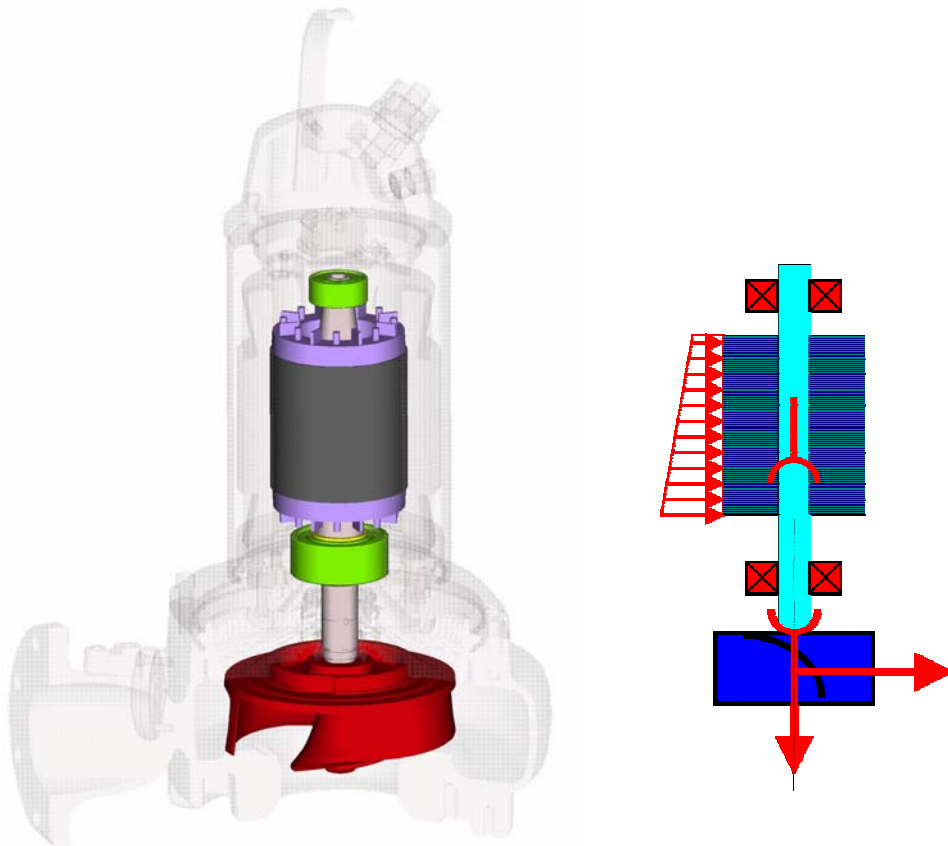


## Shaft and Bearing Calculations

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**Abstract.** Years of field experience and laboratory testing, together with advanced calculations, form the basis for development of Flygt methods and computer programs used to analyze and dimension the rotating system of a pump or mixer. Bearings are one of the key components that determine service intervals for Flygt pumps and mixers. Therefore, knowledge and understanding of the variety of factors that influence a bearing's lifetime is of utmost importance. A correctly dimensioned shaft is fundamental to ensuring smooth and trouble free running with no disturbances from natural frequencies, large deflections or fatigue failures. Knowledge of unpredictable causes of failure such as penetrating fluid in the bearing or assembly damage to the shaft, are also an important part in the design of a sturdy and reliable product. Proven models of geometry as well as load cases that cover the unpredictable directions of tolerances and related forces are necessary. The amount of data needed (and generated) in shaft and bearing calculations is huge. Consequently, it is impossible to analyze all combinations of impellers/propellers, volutes, drive units and running conditions. What *is* important is the performance of the product and our ability to create a robust product that fulfills the customer's demands. Other important issues that are not addressed in this publication, but which are vital to creating well designed systems, are proper installation with favorable inlet conditions, favorable duty points, secure anchoring, etc.



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## 1. Introduction

Calculating hydraulic, dynamic and electrical forces and assessing the environment in which the product will work is a very complex process. Through years of field experience and experimentation in research laboratories, Flygt has developed computerized methods for calculating forces and the outcome of the forces applied to the rotating system of a pump or mixer. This is essential for the design of the rotating system including shaft, rotor, seals and impeller/propeller, and for the selection of bearings with adequate bearing life.

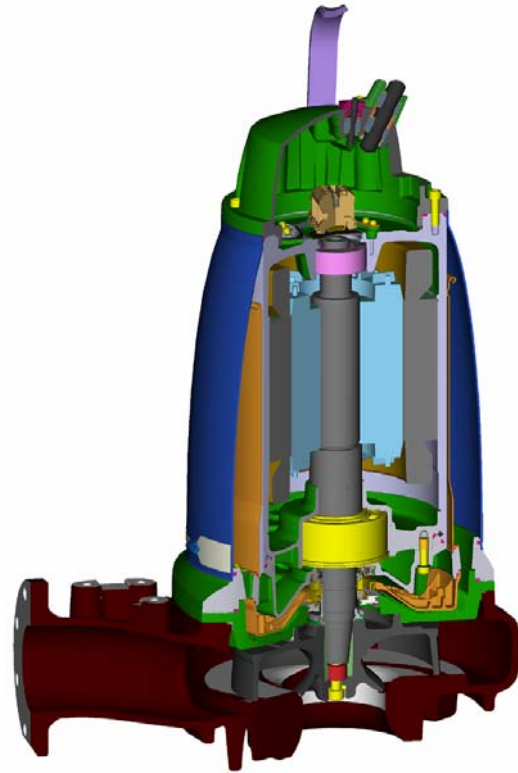
Bearings are one of the key components that determine the service interval for rotating machinery; they can even limit the life of the machine. Therefore, knowledge and understanding of bearings are important to both designers and users. In selecting the bearing to be used, one must consider a variety of factors:

- ◆ Loads, maximum and minimum, static and dynamic.
- ◆ Speed and running pattern.
- ◆ Stiffness.
- ◆ Temperature levels and gradients, heat conduction.
- ◆ Lubrication: viscosity, stiffness, durability.
- ◆ Degree of contamination. Shielding.
- ◆ Mounting.
- ◆ Maintenance.
- ◆ Lifetime and service time.

A bearing selection or analysis is futile without rotor dynamic data from the shaft and other rotating parts. A correctly dimensioned shaft is fundamental and there are many criteria that need to be fulfilled:

- ◆ Magnetic behaviour, especially for 2-pole motors.
- ◆ Fatigue durability.
- ◆ Limited deflection for impeller, seal and rotor positions.
- ◆ No natural frequencies of the rotating system close to rotational speed.
- ◆ Low impact on the surroundings regarding vibration.
- ◆ Limited angular displacement at bearing positions.
- ◆ Temperature conduction to ensure suitable bearing temperature.

Some of the items above improve with a larger shaft and some with a smaller shaft.



**Figure 1.** A midrange sewage pump showing the typical layout of a Flygt product.

To ensure the long life of a product, many factors have to be taken into account. A poor installation with unfavorable inlet conditions, unfavorable duty points, bad anchoring, etc., can cause structural disturbances that may be detrimental. Natural frequencies in the structure from poorly designed support of the pump, pipes or valves often cause high vibration levels. Those matters are not covered in this booklet; see Reference 1 for additional information.

This booklet describes some of the knowledge and experience that Flygt has acquired, including precautions to observe when designing and analysing the shaft and bearings.

## 2. Causes of failure

In order to properly dimension the rotating system, potential causes or modes of failure have to be determined. Most failures are due to unpredictable factors. The cure for these failures is to create a robust design that can cope with the unknown. Predictable failures or lifetimes can be calculated, although many parameters will vary and statistical considerations have to be taken into account.

### 2.1 Shaft

The shaft is designed to have an infinite life. This will be the case unless the shaft is overloaded or damaged.

#### 2.1.1 Fatigue crack

The cause of a broken shaft is usually fatigue. A crack may start at a stress concentration at a keyway or a sharp radius, or in rare cases, from a material impurity. Flaws in the surface of a shaft, such as scratches, indents or corrosion, may also be the starting point of a fatigue crack. Loads that drive a crack are normally torsion loads from direct online starts or bending loads from the hydraulic end.

#### 2.1.2 Plastic deformation

Plastic deformation can only occur in extreme load cases when debris is squeezed into a radial clearance that results in large deformations.

#### 2.1.3 Shaft defects

Shaft defects that have gone unnoticed during factory checks for unbalance as well as the check during testing are very rare. On site, however, it is important to prevent damage from corrosion, indents etc., by proper handling.

### 2.2 Joint

A poorly designed joint between the shaft and impeller can be detrimental for the system during both installation (high mounting forces, impacts) and operation.

#### 2.2.1 Loose connection

A loose connection can cause impacts that generate high loads and damage to the shaft and/or impeller. It is essential to tighten the bolt(s) correctly with the proper torque and lubricate with appropriate lubricant. With conical joints, lubrication is of great importance to the conical sleeve or the conical part of the impeller as it rises on the shaft and creates the necessary pressure to enable the holding force.

#### 2.2.2 Misaligned connection

This is normally hard to achieve, but it may occur with incorrect mounting.

## 2.3 Bearing

A roller bearing does not last forever. Sooner or later fatigue, wear, or lubricant deterioration will ultimately destroy the bearing's ability to function properly. Causes of bearing failure are listed below in order of likelihood.

### 2.3.1 Penetrating fluid or particles

Cleanliness is essential for good performance. Particles generate high stresses in the bearing components and create premature fatigue failure. Particles also generate wear that shortens life. Particles, especially light alloy particles like zinc, may act as catalysts for the grease and create premature aging. Consequently, Nilos rings must not be used!

If a fluid enters the bearing, contact with the rolling elements will cause the fluid to act as a jet and force the grease out from the raceway. If the fluid is oil with just a tiny amount of water (0.1%), the bearing's lifetime is ruined even if it is flooded with that oil. Furthermore, the fluid may also cause corrosion.

### 2.3.2 High temperature or gradient

High temperatures due to inadequate cooling or too much generated heat can be identified (if the bearing is not totally ruined) by darkened grease and a feeling of carbon particles dissolved in oil. If synthetic oil is used, it may have polymerized and created a lacquer layer on the surface, mainly seen in oil lubrication. A bearing that has been overfilled with grease may also overheat.

If the temperature is just slightly higher than allowed, the signs are not so obvious. However, the lifetime will be reduced due to lower viscosity and faster degradation of the lubricant. Cages with plastic material will also age faster at increased temperatures. The normal temperature limit for a Flygt product is 90°C, but peak temperatures up to 120°C are allowed.

If too much heat is generated, perhaps from overload but having adequate cooling, then the gradient over the bearing can be too high, generating a preload that can destroy the bearing. A correct internal bearing clearance is vital.

### 2.3.3 Incorrect mounting

Improper mounting in the field causes a high amount of bearing failures. Precautions that must be taken include:

- ◆ Keep everything clean and protected from corrosion during mounting and storage.
- ◆ Do not overheat. The max temperature is 150°C for very short periods. The bearing is stabilized to a certain temperature and temperatures above this may alter the

internal structure of the material, although it takes some time. The cage may also be damaged if subjected to overheating.

- ◆ Use suitable tools. Avoid mounting in a way that the mounting force goes through the rolling element which may cause indentations and destroy the bearing.
- ◆ Use the correct grease or oil and do not overfill. Bearing lifetime is calculated with a specific lubricant. Note that different greases or oils should not be mixed as the result may be disastrous.
- ◆ Use the correct bearing as specified by Flygt. Standard bearings may have improper lubricants, clearances, angles or be unsuitable in other ways.
- ◆ Always follow the Flygt guidelines.

#### 2.3.4 Shock loads from handling

Shock load may be transmitted from mounting of other parts, i.e. impeller, or during transport and installation.

#### 2.3.5 Vibration at standstill

During operation, the bearing can withstand rather high vibration. Normally 25 mm/s rms does not have a significant effect on the lifetime. The grease though may be "shaken" away depending on installation and grease type. At standstill, the allowable levels are much lower as false brinelling may occur at levels as low as 7 mm/s rms. Poor installation of piping, stands, etc., may cause harmful vibration levels in stations with more than one pump.

#### 2.3.6 Low load with stiff grease

Surfing occurs when the rolling elements ride irregularly (not spinning) in the unloaded zone and are forced to spin in the loaded zone, and thereby create a skid mark in the rolling surface. Preloaded bearings by weight or springs can overcome this. Oversized bearings have higher risk of surfing.

#### 2.3.7 Electrical current

Large electric motors and motors driven by low quality frequency drives may suffer from stray current. Other equipment such as welding machines may also create stray current. Some of Flygt's larger motors have a ceramic coating on the outer ring of the bearing to avoid this problem. The current does not always ruin the surfaces; however, the grease can be destroyed in a short period of time and create a failure.

#### 2.3.8 Fatigue

The calculated lifetime of a bearing, defined as the time to the first sign of pitting or scaling (not a total failure), is dependent on the number of revolutions, the load and the lubrication. Initial fatigue damage,

in the form of small cracks, results from cyclic shear stresses that are highest just below the surface. The cracks then propagate up to the surface. As the rolling elements pass over these cracks, small fragments will finally break off (scaling). The calculated lifetime varies in a statistically known probability according to the Weibull distribution gathered from many tests.

#### 2.3.9 Defective bearing, housing or shaft

Defective bearings are extremely rare nowadays. Shafts and the bearing housings from the factory are also very rarely out of tolerance, and it is even more unlikely that they will be shipped to a customer. It is very important that the mounting is correct so that uneven tightening of bolts does not ruin the tolerance.

### 2.4 Failure detection

Depending on the customer's philosophy regarding service and maintenance, there are several ways to act on a failure indication. The simplest approach is to operate until total disaster. Another approach is to perform preventive maintenance at recommended intervals to (hopefully) avoid any failure. The most advanced approach is to use sensors and analyzing tools to spot early signs of faults and act when needed. Listed below are possible indications:

- ◆ **Noise.** You can hear severe damage. You can amplify the noise by placing a screwdriver or a wooden rod in contact with the machine and your ear. Be aware though, that all bearings create some noise, especially during start up while the clearance is still at its nominal value. When the machine is warm, the noise is normally lowered.
- ◆ **Vibration.** Vibrations caused by loose parts contain many frequencies and can be hard to separate from the noise of cavitation and debris in pumped media. Vibration caused by a defective bearing can only be detected with sophisticated tools, and even then it is difficult to identify if you do not have the vibration pattern of a good operating condition with the actual bearing for comparison. Condition monitoring (with acoustic emission, see-technique, etc.) is a good tool if you are able to measure the vibration close to the bearing and if you set up individual limits for different running conditions. Vibration from a damaged bearing close to failure or a loose part may be spotted by simpler vibration measure devices. This can be used to stop the machine before total breakdown.
- ◆ **Temperature.** Temperature measurements are unable to detect defective bearings. In some

cases you may see a temperature rise just before breakdown. In rare cases, trend measurements of temperatures may sometimes give information that indicates a malfunction due to increased heat generation in the bearing.

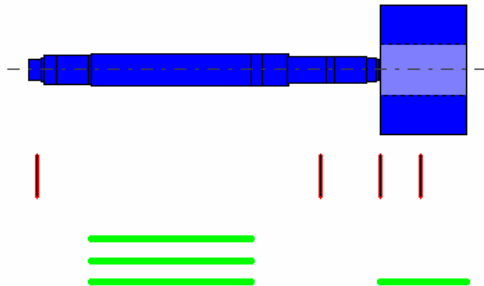
### 3. Geometry Modelling

Simplified geometrical models of the rotating system are needed to perform the calculations. Only the mechanical model is discussed here. Heat transfer models, structural models involving the entire pump and connected items, and electrical models are not mentioned in this booklet.

#### 3.1 Shaft

The geometry of the total shaft is made of pipe segments with or without a hole. No asymmetrical parts are allowed. Each segment has its modulus of elasticity and density. Each segment can also have an extra added mass and diametrical moment of inertia to simulate mass with no stiffness. Figure (3.1) shows a modelled shaft with concentrated and distributed load marked.

C3231/765-60Hz 600V st 0 I/s



**Figure 3.1.** A shaft model as seen in program AXEL with marked loads.

#### 3.2 Rotor

The rotor has to be modelled as a shaft part as above. The mass and inertia of a rotor can easily be calculated or measured. The stiffness of the rotor is more difficult to get. To measure the stiffness, it is important to have deflections in the region of an actual rotor. Deflections that are too small, like the ones you get by pounding on the rotor with a hammer and then measuring the frequency response with an accelerometer, will result in a stiffness that is too high. From extensive measurements at Flygt (rotors from 2 kg to 2000 kg), equation (3.2) gives the stiffness equivalent diameter  $de$ .

$$de = d \cdot \left( 1 + \frac{Eer}{Ea} \left( \frac{Eesp}{Eer} \cdot \left( \frac{dy}{d} \right)^4 + \left( 1 - \frac{Eesp}{Eer} \right) \cdot \left( \frac{di}{d} \right)^4 - 1 \right) \right)^{.25}$$

$de$  = stiffness equivalent diameter (3.2)

$d$  = shaft diameter

$di$  = rotor core diameter

$dy$  = rotor diameter

$Ea$  = shaft modulus

$Eer$  = equivalent core modulus

$Eesp$  = equivalent slot modulus

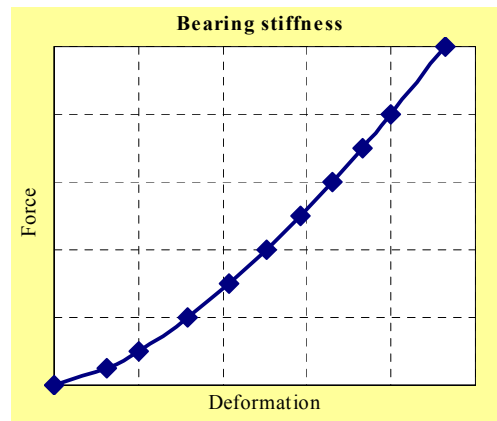
This is calculated in the Flygt program ROTOR.

#### 3.3 Impeller/Propeller

The stiffness of the impeller is easier to model since it is so stiff that it is accurate enough just to assign a high value here. The mass and inertia are normally taken from the cad part and the lack of symmetry is normally of less importance.

#### 3.4 Bearing

In most cases, it is adequate to model the bearing as a stiff support in Flygt products. If the bearing housing is radially weak or the bearing stiffness is of importance, this can be simulated by a spring with suitable stiffness. In the case of a bearing that can take some bending and the shaft angle is more than about 4 angular minutes, this technique is to be used. The stiffness of a bearing is load dependent. Therefore, in linear analysis some runs are necessary to achieve the correct stiffness. See Figure (3.3).



**Figure 3.3.** The non-linear behavior of a typical ball bearing.

### 4. Loads

Different loads act on the rotating system and there is a lot of work and experience involved in achieving accurate forces. Moreover, without accurate forces sophisticated rotor dynamic

analyses are of little value and lifetime calculations are futile.

#### 4.1 Hydraulic forces

The calculation of hydraulic forces is part of a package containing an extensive set of computer programs used in the design and calculation of the hydraulic components. The programs have been verified by extensive testing in Flygt's RD&E laboratories. See Reference 3. To investigate further details of flow pattern, using CFD software is helpful, as described in Reference 4.

##### 4.1.1 Radial pumps

The axial force from an impeller is normally directed towards the inlet as the pressure is lower there than on the back of the impeller, especially for open impellers. Sometimes large flows may create axial forces that shift direction, a situation to be avoided since it may have a harmful influence on the bearings.

Radial forces caused by the impeller and volute are at their minimum at nominal flow since the pressure distribution and internal flow are most favorable there. See Figure (4.1.1). Since these forces are the dominating ones in many cases, it is very important to ensure that the duty point is satisfactory. The frequency of the dynamic force is determined by the number of vanes. Single vane impellers create large dynamic forces.

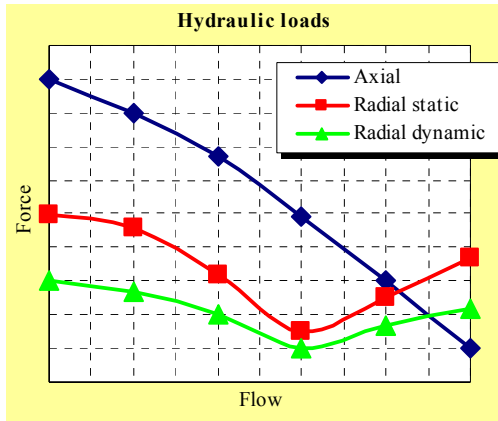


Figure 4.1.1. The radial hydraulic forces from the impeller reach a minimum at nominal flow.

##### 4.1.2 Axial pumps and mixers

For mixers and axial pumps without volutes, the radial forces are smaller and normally neglected. The axial force or thrust, may cause a bending moment if there is a skewed inflow. Equations (4.1.2) give:

$$Mb = F \cdot A \cdot \frac{kd}{2} \text{ if } > 2 \text{ blades}$$

$$Mb = F \cdot A \cdot kd \text{ if } 2 \text{ blades} \quad (4.1.2)$$

$$A \approx \frac{D}{3} \text{ for a large propeller}$$

$Mb$  = bending moment

$F$  = thrust

$A$  = radius to axial force on blade

$Kd$  = skew factor

$D$  = propeller diameter

A skew factor of 1 indicates that one blade has twice the normal force in one position and no force in another. A normal value to use for mixers is 1/3.

#### 4.2 Motor forces

The value of the magnetic effects in the motor as well as torque and speed data during start are obtained from software handling motor calculations and from Flygt's RD&E laboratories.

##### 4.2.1 Axial

The axial force from skewed rotor bars is normally negligible compared to other forces.

##### 4.2.2 Radial

The magnetic field surrounding the rotor in the motor produces a radial force if any eccentricity is present. The magnetic force is expressed as:

$$F = dmp \cdot C \cdot \varepsilon \cdot f(\varepsilon)$$

$$\varepsilon = \frac{e+u}{Lg} \quad (4.2.2)$$

$$C = K \cdot L \cdot D$$

$F$  = force

$dmp$  = damping

$C$  = unbalanced pull

$\varepsilon$  = relative eccentricity

$f(\varepsilon)$  = non-linear part

$e$  = eccentricity

$u$  = deflection

$Lg$  = air gap

$K$  = factor including magnetic flux parameters

$L$  = length of rotor

$D$  = rotor diameter

The non-linear behavior causes the force to be very large at high eccentricities. If  $\varepsilon$  is less than 0.3, the non-linear part is negligible, and if it is less than 0.5, it is small. The value  $K$  can be estimated as 0.2 MPa for 2-pole motors, and as 0.4 MPa in all other cases. The damping value is normally smaller than 0.3. The air gap to use is the magnetic gap that at small eccentricities is equal to the physical gap. However, at high magnetic flux, as at high

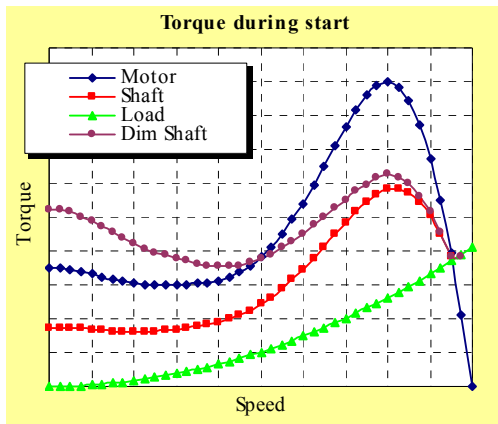
eccentricities and small physical gaps, the magnetic gap is larger.

The magnetic force is divided into two parts: One is dependent on eccentricity ( $e$ ) and the other depends on the displacement, like a spring but with force and displacement in the same direction. That is, if the air gap is smaller on one side than on the other, the magnetic flow will pull the rotor in the direction of the smallest gap. The Flygt program ROTOR handles this calculation.

To get both the static and the rotating eccentricity, many tolerances have to be added together. To get a representative value of the eccentricity in both ends of the rotor, the tolerances with their maximum values are added to form a quadratic sum. This is done with the Flygt program EXCENT.

#### 4.2.3 Torque

The dimensioning torque for a typical Flygt product comes from direct-on-line start. Figure (4.2.3) shows torque curves from a start.



**Figure 4.2.3.** The torque curves during direct on line start.

The motor curve and the shaft curve are the nominal ones; the actual curve has disturbances of an electrical nature that give high peaks during the start sequence. The dimensional shaft curve has been modified to cope with this. The load curve is quadratic which is normal for a pump curve. Equation (4.2.3) shows how the nominal shaft curve can be calculated:

$$M_a = M_l + \frac{i^2 \cdot J_l}{J_m + i^2 \cdot J_l} \cdot (M_m - M_l) \quad (4.2.3)$$

$M_a$  = shaft torque  
 $M_l$  = torque of load  
 $i$  = gear ratio (if any)  
 $J_l$  = polar inertia of load  
 $J_m$  = polar inertia of motor

This is done for each speed during start and the highest value of the dimensional shaft curve is used. The Flygt program MOMENT is used for this.

### 4.3 Mass forces

If the rotating system is exposed to high vibration levels, then mass forces are to be added. If the vibration levels emerge from the rotating system, then the mass forces may cause lower reactions, all depending on the dynamic behaviour of the entire installation with housing, anchoring, piping, etc. These phenomena are not covered here.

#### 4.3.1 Weight

The static force from the weight is comparably low and is beneficial for the bearing in order to ensure a minimum load.

#### 4.3.2 Unbalance

The unbalance of the rotor and impeller causes radial forces when the machine is rotating. The maximum allowed unbalance is used in the calculation. The force is calculated as:

$$F = Ub \cdot \omega^2 \quad (4.3.2)$$

$F$  = force

$Ub$  = unbalance

$\omega$  = rotational speed

This force is distributed over the length of the part, and if it acts in two planes (dynamic balancing), the values are added to form a vector sum.

The unbalance value is normally according to ISO 1940 G6.3, but higher values are permitted for components if the product in another design contains rotating components of larger mass or inertia. Higher values can also occur if the rotating parts have a minor effect on the product or its surroundings. The allowed unbalance value is divided between two planes if motivated by the geometry of the part.

### 4.4 Thermal influence

A thermal analysis provides the temperature distribution needed to get the elongation of the shaft and the temperature and temperature gradient over the bearings. Elongation is used for the calculation of the axial spring force of axially preloaded bearings or the required tolerance for the expansion. Bearing temperatures are required for the calculation of correct fit and clearance, as well as the lifetime calculation. The thermal condition may even require a larger size of the bearing due to the heat transfer capabilities rather than the load carrying capabilities. Thermal calculations are made using the Flygt program HEATS SP that is



described in Reference 2 and the outcome is shown in Figure (4.4).

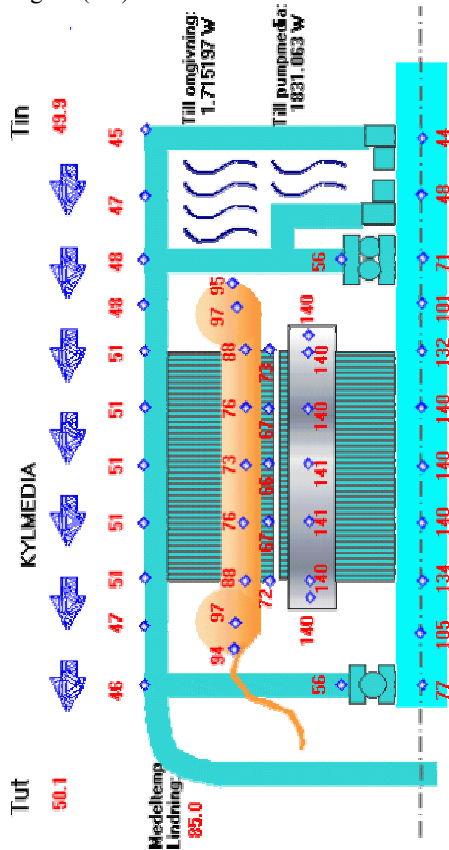


Figure 4.4. An example of the temperature distribution of a Flygt motor.

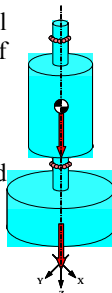
## 5. Load cases

From the calculated forces, one can define load cases that represent different types of analyses. In this section, the forces are collected and applied to get a base for further calculation. This is a generalized description; there may be more forces not mentioned here from such things as gears, for example.

### 5.1 Axial

Normally there are no dynamic axial forces of importance. Forces of importance are:

- ♦ Axial hydraulic force or thrust.
- ♦ Weight (if vertical)
- ♦ Spring forces from preloaded bearings.



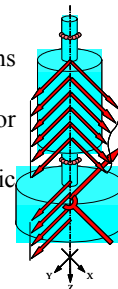
## 5.2 Radial

There are three main types of radial loads: Static, rotating with shaft, and rotating with blade pass frequency. To be accurate, the last force is not truly rotating, but for the purpose of shaft analysis it is a sufficient approximation. Several of these loads differ from one unit to another and their direction is undeterminable. To cope with this variability, we use maximum values (of tolerances, for example) and add them to form a quadratic sum. In addition, they should be combined to give a representative picture of a typical load case. The general procedure at Flygt is to have 90° between two consecutive loads and always between the two major ones, as shown in the figures. This technique ensures that most units have lower load levels than the calculated ones, and it also gives a standardized way of handling the unknown factors.

### 5.2.1 Static

Forces of importance (their directions are shown in the picture):

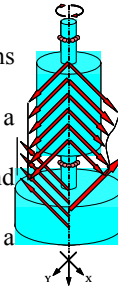
- ♦ Hydraulic force from volute or torque from propeller.
- ♦ Magnetic pull from an eccentric rotor.
- ♦ Magnetic "spring".
- ♦ Weight (if horizontal)



### 5.2.2 Rotating

Forces of importance (their directions are shown in the picture):

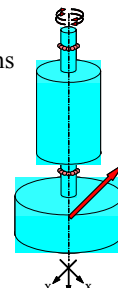
- ♦ Hydraulic force from impeller (if a single vane impeller).
- ♦ Unbalance from rotor and impeller/propeller.
- ♦ Magnetic pull from eccentricity by a bent rotor or play in bearings etc.
- ♦ Magnetic "spring".



### 5.2.3 Blade pass

Forces of importance (their directions are shown in the picture):

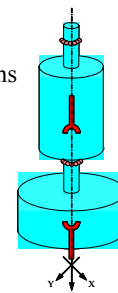
- ♦ Hydraulic force of impeller.
- ♦ Magnetic "spring".



## 5.3 Torsion

Forces of importance (their directions are shown in the picture):

- ♦ Motor torque.



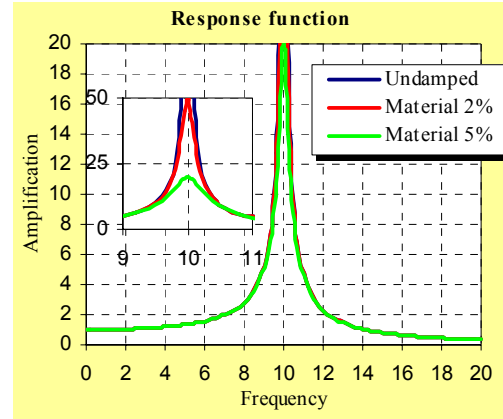
- ♦ Polar inertia during acceleration.
- ♦ Hydraulic torque.

## 6. Shaft calculations

To handle the radial loads and rotor dynamics, we normally use a software package, AXEL, developed at Flygt. The program uses a finite difference method. In the calculation, the symmetrical shaft is split into increments. The variables (displacement, angular displacement, moment and shear force) at one side of each increment are expressed in terms of the same variables on the other side of the increment in conjunction with an incremental transfer matrix. Loads of different types and bearings have their own transfer matrices in which the increment size is zero. All matrices are multiplied to obtain a total transfer matrix. This process is repeated successively; that is, the matrix is updated for each increment and each load. The matrix can then be assembled in accordance with the boundary conditions at the end of the shaft (if internal boundaries, then sub matrices have to be saved for later use). The remaining boundary variables are solved, and then all unknown variables in between the boundaries can be evaluated by means of the same technique based on the updated transfer matrix (using the saved sub matrices if internal boundary conditions exist). Natural frequencies are found when the determinant of the total matrix shift sign while stepping the frequencies. Since natural frequencies are load independent and no damping is involved, the matrices are somewhat simpler. The torsion natural frequencies are also handled by this software. For calculation of deflections, stresses, etc., each load case, with its rotational speed, has to be calculated with the actual shaft rotation. Only in the load case that rotates with shaft speed are both load and shaft rotation equal. Axial and torsion calculations are comparatively simpler and can consequently be made with simpler tools.

### 6.1 Natural frequencies

It is essential to avoid natural frequencies that coincide with the frequencies of disturbance forces since the dynamic deformations are amplified dramatically close to a natural frequency. The influences of the damping set the limit close to natural frequencies and are of limited importance elsewhere, as shown in Figure (6.1) of the response function.



**Figure 6.1.** The response functions around a single critical frequency for different damping. Max value for 2% damping is 50 and for 5% 20.

Damping values for steel are around 1%, and the damping for the entire rotating system at normal displacement is higher, roughly up to 5%. With pumps there is also some viscous damping (reaction force linear to velocity). The relation is expressed in Equation (6.1).

$$\Phi = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_{cr}^2}\right)^2 + \eta^2}} \quad (6.1)$$

$$c = \eta \cdot \frac{K}{\omega}$$

$\Phi$  = amplification

$\omega$  = frequency

$\omega_{cr}$  = critical frequency

$\eta$  = material damping

$c$  = viscous damping

$K$  = Static stiffness at position of damping

The peak is flattened in a real application due to non-linearity and the picture is more complex due to the influence of other mode shapes and frequencies.

#### 6.1.1 Axial

Axial natural frequencies are normally very high and of no practical interest.

#### 6.1.2 Bending

Bending natural frequencies are hard to calculate by hand, and furthermore these are of most interest as they normally are the lowest ones. In the program AXEL, a frequency range is scanned and the selected amount of the lowest natural frequencies is calculated. Flygt products have at least a margin of 30% between the rotational speed and the first natural frequency of the rotating

system. The presented natural frequencies are calculated with load and shaft rotating with equal speed.

### 6.1.3 Torsion

Torsion frequencies are calculated with the AXEL program, but as a rough check of the first mode Equation (6.1.2) may be used

$$\omega_{kr} = \sqrt{\frac{G \cdot K}{L} \cdot \left( \frac{1}{J_M} + \frac{1}{J_L} \right)} \quad (6.1.2)$$

$$G = \frac{E}{2 \cdot (1 + \nu)}$$

$\omega_{cr}$  = critical frequency, torsion

$K$  = torsion stiffness

$L$  = length

$G$  = torsion modulus

$E$  = modulus of elasticity

$\nu$  = Poissons ratio

$J_m$  = inertia motor

$J_l$  = inertia load (impeller)

## 6.2 Reactions

To correctly design and dimension the boundary to the rotating system (bearings, housings, etc.) the reactions are needed.

### 6.2.1 Axial

The axial reaction is mainly static during run and is carried by the main bearing arrangement and its housing, except for spring preload if any. The load is the sum of forces from Section (5.1).

### 6.2.2 Radial

The radial reactions are a bit more complex as well as the load case shown in Section (5.2). Each radial load case has to be calculated and the reactions then have to be totalled. For bearing calculation, the static load case should be added to the sum of magnitudes for the dynamic load cases in the special way described in Section (7.1).

### 6.2.3 Torsion

Torsional reactions are taken by the stator and its housing, and the load is shown in Figure (4.2.3) with the addition of the electrical disturbances during start.

### 6.2.4 Global reactions

The rotating system is one of the major causes of reactions that are taken by the structure holding the pump or mixer. The axial reactions are those from axial loads from thrust or inlet conditions and disturbances and weight. The radial reactions from unbalances and unbalances caused by rotating displacement can be achieved from an unbalance calculation by the AXEL program. The program gives the unbalance data at start position of the

shaft and they can be used to calculate forces acting on the base or as a part in estimating vibration levels. To get the forces from unbalance data, equation (6.2.4) can be used

$$F_x = \omega^2 \cdot Ub_x$$

$$z_x = \frac{DUB_{xz}}{Ub_x} \quad (6.2.4)$$

$F_x, F_y$  = forces in x and y direction

$z_x, z_y$  = position of forces

$\omega$  = rotational speed

$Ub_x, Ub_y$  = unbalance

$DUB_{xz}, DUB_{yz}$  = moment unbalance

More reactions that are global emerge from the created flow and pressure from the hydraulic end. The rotation reactions are those mentioned in the previous section and they give a start jerk that has to be considered. In this section, the focus on the rotating system is a bit vague and only the matters closely connected to the shaft calculation are covered with some depth.

## 6.3 Displacement

Even if a shaft can cope with rather large deflections, other components such as seals may fail if the deflections or angular deformations are too large.

### 6.3.1 Axial

The axial displacement is calculated according to Equation (6.3.1)

$$\delta = \int_L \left( \frac{F \cdot dL}{A \cdot E} + \alpha \cdot dL \cdot \Delta T \right) \quad (6.3.1)$$

$\delta$  = displacement

$F$  = force

$L$  = length

$E$  = modulus of elasticity

$A$  = cross section area

$\alpha$  = thermal expansion coefficient

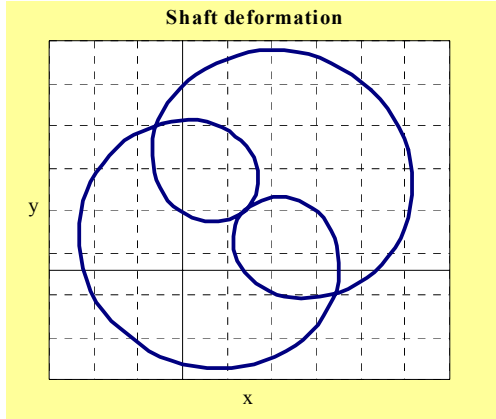
$\Delta T$  = temperature change

The integration or sum of parts with different temperatures, geometries or loads goes from the axial boundary condition that is normally fixed at the main bearing and axially free elsewhere. The axial displacement shall not cause any rotating part to interfere with the surroundings or be large enough to create problems for support bearings, seals, etc.

Parts of different geometry or load may be calculated separately and added together.

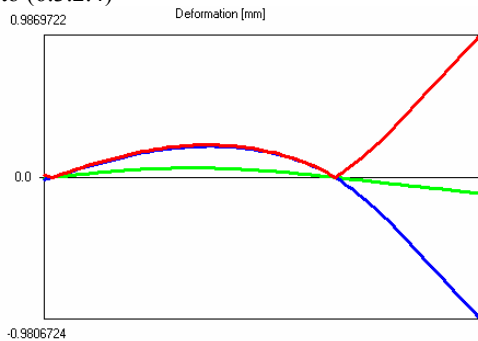
### 6.3.2 Radial

Radial deflection is calculated by the AXEL program. With two dynamic load cases, the deflection, if plotted for some position on the shaft, can look a bit odd as shown in Figure (6.3.2.1).

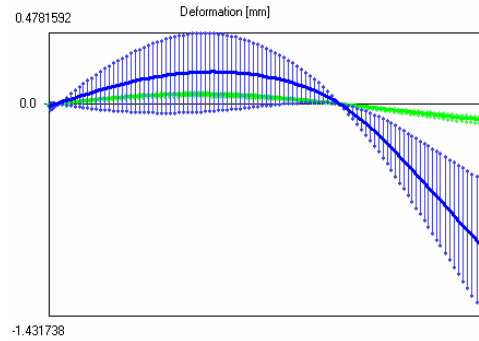


**Figure 6.3.2.1.** The deformation pattern of static, rotational and blade pass (3 blades) load cases with equal amplitudes and equal phase.

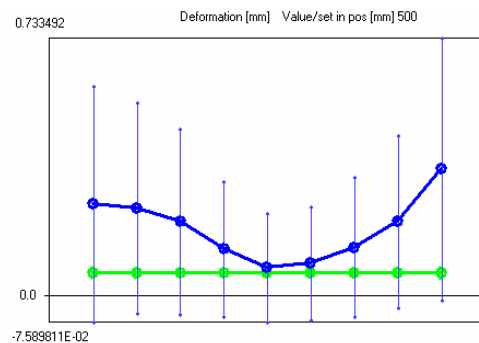
The detailed pattern is of less importance, and furthermore the phase is normally unknown. To cope with this, the amplitude of one of the two dynamic load cases is added to the amplitude of the other to form a maximum dynamic deflection. An example from AXEL is shown in Figures (6.3.2.2) to (6.3.2.4)



**Figure 6.3.2.2.** The deformation of the static run at one duty point. Red is magnitude and green and blue are x and y-data. Data from start to end of shaft.



**Figure 6.3.2.3.** The deformation of the static run with the sum of the two dynamic runs superimposed at one duty point. Data from start to end of shaft.



**Figure 6.3.2.4.** The deformation of the static run with the sum of the two dynamic runs superimposed in one position of shaft and at different duty points.

### 6.3.3 Torsion

Torsion displacement is of small practical interest in normal pumps with their relatively short shafts. If calculated, the Equation (6.3.3) gives:

$$\varphi = \frac{M_t \cdot L}{G \cdot K} \quad (6.3.3)$$

$$G = \frac{E}{2 \cdot (1 + \nu)}$$

$\varphi$  = angular motion

$K$  = torsional stiffness

$L$  = length

$G$  = torsion modulus

$E$  = modulus of elasticity

$\nu$  = Poissons ratio

$M_t$  = torque

### 6.4 Stresses

The static and dynamic stresses of the shaft are needed to design the shaft accurately with respect to plastic deformation and fatigue.

#### 6.4.1 Bending and Axial

The axial stress is calculated according to equation (6.4.1.1) and is normally comparatively small.

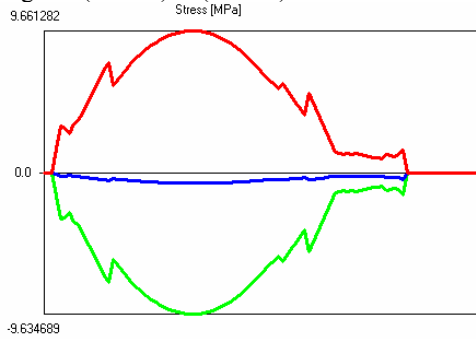
$$\sigma_d = \frac{F}{A} \quad (6.4.1.1)$$

$\sigma_d$  = axial stress

$F$  = force

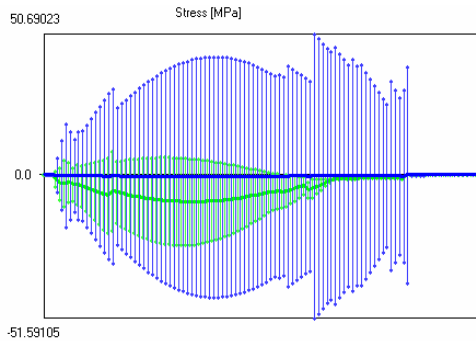
$A$  = cross section area

The bending stress is calculated by AXEL and here the rotating loads are static relative to the shaft. To calculate the dynamic part, the static run and the blade pass run have to be added. The sum is created by adding the magnitude of the blade pass run to the static run. Note that this technique is only valid for calculation of maximum values and values to use for infinite life calculations. To get stresses to use in order to estimate crack propagation as a function of time, the load cases cannot be added in this way. An example from AXEL is shown in Figures (6.4.1.1) to (6.4.1.3)

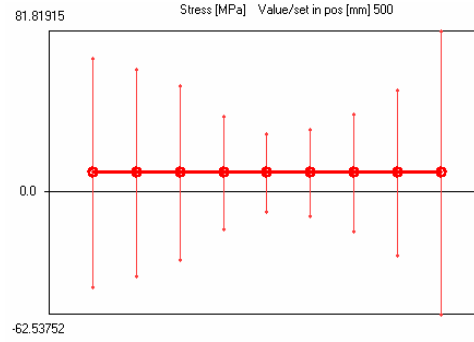


**Figure 6.4.1.1.** The bending stress of the rotating run in one duty point. Red is magnitude and green and blue are x and y-data. Data from start to end of shaft.

In the data shown in Figure (6.4.1.1), the magnitude of axial stress in the different positions of the shaft are to be added to form the maximum normal stress.



**Figure 6.4.1.2.** The bending stress of the rotating run with the sum of the static and blade pass runs superimposed in one duty point. x and y data only. Data from start to end of shaft.



**Figure 6.4.1.3.** The bending stress of the rotating run with the sum of the static and blade pass runs superimposed in one position of shaft and in different duty points. Magnitude only.

All stresses shown and mentioned so far are nominal, that is, without the influence of stress concentration factors from sharp corners, keyways, etc. To get maximum tensile stresses, the concentration factors are needed as shown in Equation (6.4.1.2).

$$\sigma_1 = \sigma_d \cdot K_d + (\sigma_{bstat} + \sigma_{brot} + \sigma_{bbp}) \cdot K_b$$

$$\sigma_1 = \text{max tensile stress} \quad (6.4.1.2)$$

$\sigma_d$  = axial stress

$\sigma_{bstat}$  = static bending stress

$\sigma_{brot}$  = rotating bending stress

$\sigma_{bbp}$  = bending stress from blade pass

$K_d$  = stress concentration, axial

$K_b$  = stress concentration, bending

Stress concentration factors give high local stresses that may yield the shaft material locally, normally without harm. More harmful is their ability to form a starting point for a crack. Stress concentrations are normally lower than 3. If higher than 5, then fracture mechanics are more appropriate. Stress concentration factors can be found in many publications or can be determined with finite element analysis.

## 6.4.2 Torsion

The shear stress from torsion is calculated from Equation (6.4.2).

$$\tau_v = \frac{M_v}{W} \quad (6.4.2)$$

$$\tau_1 = \tau_v \cdot K_v$$

$\tau_1$  = max shear stress

$\tau_v$  = shear stress

$M$  = torque

$W$  = section modulus

$K_v$  = stress concentration, shear

The stress concentrations for torsion are normally lower than for bending.

#### 6.4.3 Combination of stresses

To get an effective stress value to compare with material data, the von Mises stress criteria is best suited for shaft materials.

$$\sigma_e = \sqrt{\sigma^2 + 3 \cdot \tau^2} \quad (6.4.3)$$

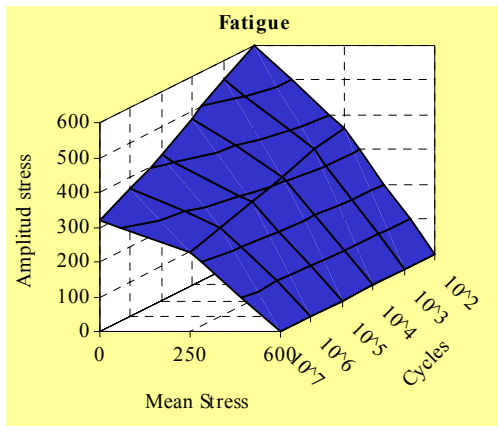
$\sigma_e$  = von Mises stress

$\sigma$  = tensile stress

$\tau$  = shear stress

### 6.5 Fatigue analysis

The method presented here is based on the construction of an idealized and linearized Haigh diagram, together with a Wöhler diagram (linearized as the logarithmic amplitude stress versus logarithmic number of cycles). Picture (6.5) shows this in a 3-D graph.



**Figure 6.5.** The combined and linearized Haigh and Wöhler diagram.

The linearization is essentially not a practical limitation since fatigue data do not normally have the highest degree of accuracy. The calculations are handled by the Flygt program FATIGUE.

#### 6.5.1 Material

To create a Haigh diagram, the fatigue data of the material is needed, either from material specification or from tests. The parameters are:

- $\sigma_b$  ultimate stress.
- $\sigma_s$  yield stress.
- $\sigma_u$  fatigue stress limit, alternate load at  $10^{N_m}$  cycles.
- $\sigma_{up}$  fatigue stress limit (amplitude) pulsating load at  $10^{N_m}$  cycles.
- $A_n$  Neuber radius.

If data does not exist, it may be estimated in a variety of ways such as from tables or from rules of thumb.

#### 6.5.2 Reductions

If the fatigue data is taken from a test specimen, and not the statistically accurate tests of the actual component, then reductions have to be made. Reductions are made with three factors:  $C_l$ ,  $C_d$  and  $C_s$  due to load, dimension and surface. If there is a stress concentration, the reductions are made with the help of a stress concentrations factor  $K_t$  and fatigue factor  $K_f$  and the dimension factor is then of less importance. The factors are described in more detail below:

$C_l$ . Reduction due to load. If fatigue data and load case do not match, this factor is used. For example, if torsion load and material data for bending do not match, then  $C_l$  is 0.58 (and  $\sigma_{up} = \sigma_u$ ). If tensile load and data for bending do not match, then  $C_l = 0.85$ .

$C_d$ . Reduction due to size. Normally close to 1. Statistically the risk of fatigue failure is larger the bigger the stressed volume is. See explanation of  $K_f$  in Equation (6.5.2.3). If no stress concentration and maximum stress is used, then Equation (6.5.2.1) may be used.

$$C_d = \frac{1 + \sqrt{\frac{2 \cdot A_n}{d}}}{1 + \sqrt{\frac{2 \cdot A_n}{10}}} \quad (6.5.2.1)$$

$d$  = characteristic size [mm]

$C_d$  is 1 according to (6.5.2.1) if the size is equal to the size of the test specimen, normally 10 mm.

$C_s$ . Reduction due to surface quality. A bad surface has more and worse starting points for a fatigue crack. Different machining and surface treatments give different  $C_s$  values, as well as the environment that the surface is exposed to.  $C_s$  may be larger than 1 from surface treatments. One example is to induce compressive stress into the surface by roller burnishing or shot peening. If a little crack of size  $a_n$  or smaller exists, then Equation (6.5.2.2) with crack depth  $a$  gives:

$$C_s = \frac{1}{\sqrt{1 + \frac{2 \cdot a}{A_n}}} \quad (6.5.2.2)$$

The fatigue factor  $K_f$  can be determined in different ways. If the stress concentration factor  $K_t$  is given and nominal stress is used, then Equation (6.5.2.3) may be used and  $C_d$  is 1 (may be smaller if stressed volume is large).

$$K_f = 1 + q \cdot (K_t - 1)$$

$$q = \frac{1}{1 + \sqrt{\frac{A_n}{K_r}}} \quad (6.5.2.3)$$

$q$  = notch sensitivity factor  
 $K_r$  = notch radius

If the stress distribution is known by FEA for example, the Equation (6.5.2.4) can give the fatigue factor.

$$K_f = \frac{1}{A_n} \cdot \int_0^{A_n} \frac{\sigma(r)}{\sigma_0} dr \quad (6.5.2.4)$$

$\sigma_0$  = max stress  
 $\sigma(r)$  = stress perpendicular to path of crack propagation

That is, mean stress to the depth of  $A_n$  divided by max stress. Here  $K_f$  is smaller than 1 and max stress is given as load.  $C_d$  is 1 (may be smaller if stressed volume is large).

### 6.5.3 Haigh diagram

In the Haigh diagram in Figure (6.5.3), five different curves can be seen.

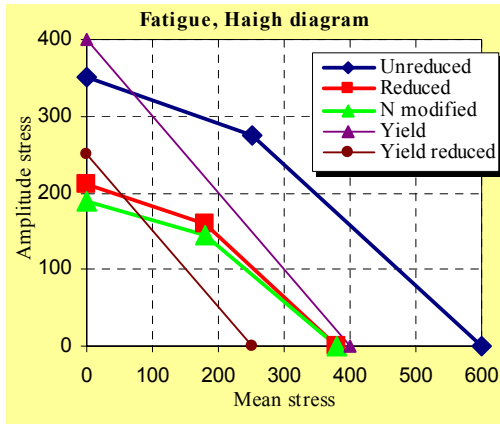


Figure 6.5.3. Haigh diagram

Curve 1, the unreduced curve is drawn from three points shown in Equation (6.5.3.1).

$$(0, \sigma_u)$$

$$(\sigma_{up}, \sigma_{up}) \quad (6.5.3.1)$$

$$(\sigma_b, 0)$$

Curve 2, the reduced curve due to load, dimension, surface and stress concentrations are drawn from three points shown in Equation (6.5.3.2).

$$(0, \sigma_u \cdot C_l \cdot C_d \cdot C_s / K_f)$$

$$(\sigma_{up} \cdot C_l / K_t, \sigma_{up} \cdot C_l \cdot C_d \cdot C_s / K_f) \quad (6.5.3.2)$$

$$(\sigma_b \cdot C_l / K_t, 0)$$

Curve 3, the curve modified due to number of cycles  $10^N$  that differ from  $10^{Nm}$  is drawn from three points shown in Equation (6.5.3.3).

$$(0, \sigma_f \cdot C_l / K_t \cdot \exp\left(\frac{N}{Nm} \cdot \log\left(\frac{\sigma_u \cdot C_l \cdot C_d \cdot C_s / K_f}{\sigma_f \cdot C_l / K_t}\right)\right))$$

$$(\sigma_{up} \cdot C_l / K_t, (\sigma_f - \sigma_{up}) \cdot C_l / K_t \cdot \exp\left(\frac{N}{Nm} \cdot \log\left(\frac{\sigma_{up} \cdot C_l \cdot C_d \cdot C_s / K_f}{(\sigma_f - \sigma_{up}) \cdot C_l / K_t}\right)\right))$$

$$(\sigma_b \cdot C_l / K_t, 0)$$

$$\sigma_f = \sigma_b \cdot \frac{1}{1 - \psi} \quad (6.5.3.3)$$

$\sigma_f$  = true ultimate stress  
 $\psi$  = reduction of area at rupture

An alternative to the true ultimate stress is to use the ordinary ultimate stress at 100 cycles. This is used in the FATIGUE program. Another alternative is to use the factor 0.9 at 1000 cycles.

Curve 4, the unreduced yield curve is drawn from two points shown in Equation (6.5.3.4).

$$(0, \sigma_s)$$

$$(\sigma_s, 0) \quad (6.5.3.4)$$

Curve 5, the reduced yield curve (local yield) is drawn from two points shown in Equation (6.5.3.5).

$$(0, \sigma_s \cdot C_l / K_t)$$

$$(\sigma_s \cdot C_l / K_t, 0) \quad (6.5.3.5)$$

### 6.5.4 Interpretation

The actual stress plotted in the HAIGH diagram should be in the area of the lower left corner, limited by the curves. For a ductile material with the mean stress lower than twice the yield stress and larger than the reduced yield curve, the actual mean stress may be moved left to the reduced yield curve. The safety factor is normally expressed as the ratio of the distance to the limiting curve and the distance to the actual stress point. If nothing explicit is mentioned, the vertical distances are used.

## 7. Bearing calculations

The aim of bearing calculations is to estimate how long a bearing will last within a specified probability. In the theoretical calculation, failure is defined as the first sign of pitting, not when the whole bearing breaks down.

### 7.1 Time equivalent load

Since there are both dynamic and static radial reactions that act on a bearing, it is necessary to form a time equivalent load to use. First, the dynamic parts are added. The magnitude of blade pass reaction is added to the rotating reaction to form the dynamic part. Then the static and dynamic parts are added as shown in Equation (7.1.1), also included in AXEL.

$$F = f_m \cdot (F_{stat} + F_{dyn})$$

$$f_m = \frac{3}{4} \cdot \left( \frac{\text{Max}(F_{stat}, F_{dyn})}{(F_{stat} + F_{dyn})} - 0.5 \right)^2 \quad (7.1.1)$$

$F$  = time equivalent force

$f_m$  = calculation factor

$F_{stat}, F_{dyn}$  = static and dynamic force

At Flygt, the bearing lifetime is calculated assuming full load and a continuously running machine. If different loads are involved during a run pattern, then Equation (7.1.2) is appropriate.

$$F = \left( \frac{\sum F_i^3 \cdot U_i}{\sum U_i} \right)^{1/3} \quad (7.1.2)$$

$F$  = time equivalent force

$F_i$  = force  $i$

$U_i$  = time fraction of force  $i$

### 7.2 Equivalent load

The relationship between axial and radial forces on a bearing affects the distribution of loads on the different rolling elements. For example, a purely axial load is to be shared by all rolling elements, but a purely radial one acts on fewer rolling elements and is more unevenly distributed. Conditions like clearance and mounting also affect this load. Normally, simple formulas from the bearing manufacturer, as in Equation (7.2), can be used although this is normally handled by bearing calculation programs. See Figure (7.3.1).

$$P = X \cdot F_r + Y \cdot F_y \quad (7.2)$$

$P$  = equivalent dynamic load

$X, Y$  = constants for different bearing types and different load cases

$F_r, F_y$  = radial and axial force

### 7.3 ISO standard 281

The ISO standard for bearing calculation has been modified in some steps since the Equation (7.3.1) was introduced in 1962. The basis for that formula was the work of Weibull, a theory of statistical endurance of metallic materials in 1936, and the work of Lundberg and Palmgren, who calculated the stresses in the volume affected by the Hertzian pressure in 1947. In 1977, the factors for reliability, material and lubrication ( $a_1, a_2$  and  $a_3$ ) were added to enhance the calculation. The latest enhancements are the introduction of fatigue load limit and influence of contamination that form the factor  $a_m$  to be used in lifetime calculations. This work by mainly Ioannidis has introduced two improvements: a fatigue stress limit and more accurate calculations of the stressed volume with local stresses, thereby including influence of surface error and contamination. Reference 5 gives the details and Reference 6 gives a "popular" version.

#### 7.3.1 Basic formula

The formula for nominal life at 10% probability of failure is:

$$L_{10} = \left( \frac{C}{P} \right)^p \quad (7.3.1)$$

$$L_{10h} = \frac{10^6}{60 \cdot n} \cdot L_{10}$$

$L_{10}$  = nominal life in rev·10<sup>6</sup>

$L_{10h}$  = nominal lifetime in hours

$P$  = equivalent dynamic load

$C$  = dynamic load rating

$p$  = exponent; 3 for ball bearings and 10/3 for roller bearings

$n$  = rotational speed, rev/min

The value of  $C$  is provided by the bearing manufacturer and achieved through a lot of testing.



A normal calculation from the SKF online catalogue is shown in Figure (7.3.1).

**SKF** [Print](#)

Equivalent bearing loads and basic rating life

Bearing: 3308 A-2ZTN9

$C_r$ , N: 60500.0

$C_0$ , N: 43000.0

$F_a$ , N: 3000

$F_r$ , N: 2000

$e$ : 0.8

$X$ : 0.63

$Y$ : 1.24

$P_r$ , N: 4980

$L_{10}$ , Mrev: 1793

$X_0$ , N: 1

$Y_0$ , N: 0.66

$P_0$ , N: 3980

Figure 7.3.1. An online calculation from SKF.

### 7.3.2 Reliability

The reliability factor is 1 for 90% reliability and it should be lowered for higher reliability: Figure (7.3.2) gives values that follow a Weibull distribution with a Weibull slope of 1.5 as adopted by ISO.

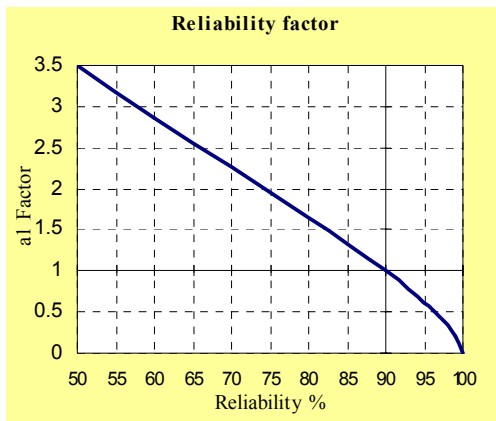


Figure 7.3.2. Reliability factor  $a_1$ .  $L_n = a_1 \cdot L_{10}$

The lifetime with altered probability is shown in equation (7.3.2).

$$L_{nh} = a_1 \cdot L_{10h} \quad (7.3.2)$$

$L_{10h}$  = nominal lifetime in hours

$L_{nh}$  = nominal lifetime with altered reliability  $n$  according to  $a_1$

$a_1$  = reliability factor

### 7.3.3 Viscosity

The required lubricant has to have sufficient viscosity in order to separate the rolling surfaces and distribute the pressure to achieve acceptable stress levels. The diagram in Figure (7.3.3.1) gives

the required viscosity as a function of bearing size and speed.

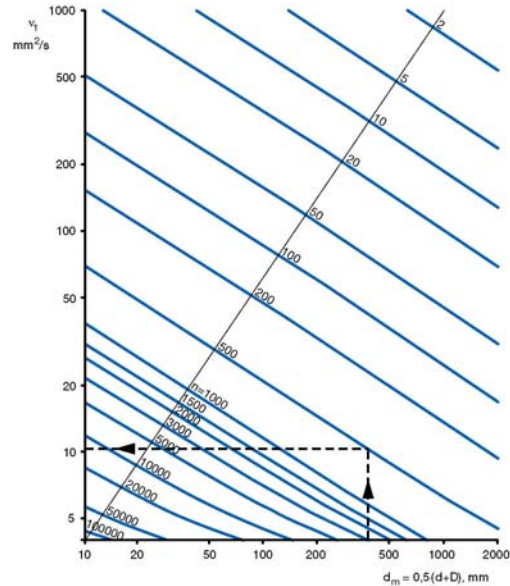


Figure 7.3.3.1. Required viscosity  $v_1$  as a function of speed and mean diameter of the bearing. (SKF)

The viscosity is highly temperature dependent and the viscosity at the running temperature is needed. The viscosity of the grease is normally given at 40°C and with a normal viscosity index the use of diagrams, as in Figure (7.3.3.2), can provide the correct figure.

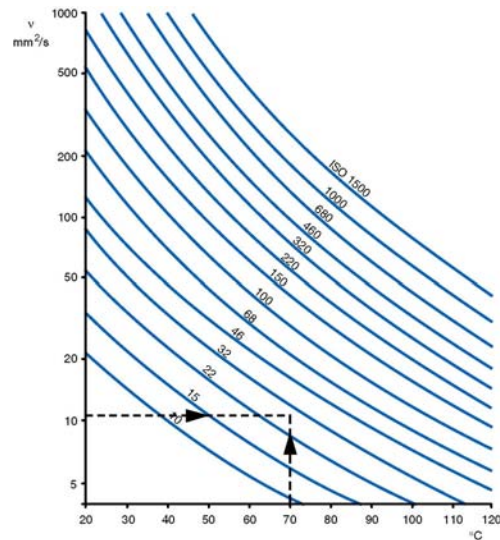


Figure 7.3.3.2. Viscosity - temperature relation for different ISO grade i.e. viscosities.

If the grease also has data for another temperature, normally 100°C, the viscosity at a given

temperature can be calculated as shown in Equation (7.3.3.1) with a log to log linearization.

$$\begin{aligned} xf(T) &= \log(T_0 + T) \\ yf(v) &= \log(\log(v^2)) \\ a &= \frac{yf(v_{T2}) - yf(v_{T1})}{xf(T2) - xf(T1)} \\ b &= yf(v_{T1}) - a \cdot xf(T1) \\ yf(v_T) &= a \cdot xf(T) + b \end{aligned} \quad (7.3.3.1)$$

$T1, T2, T3$  = temperatures in °C with ...  
 $v_T, v_{T1}, v_{T2}$  = ... corresponding viscosity  
 $T_0 = 273.15$

With the required viscosity and the actual viscosity, the viscosity relationship in Equation (7.3.3.2) can be obtained for further use in lifetime calculation.

$$\kappa = \frac{\nu}{\nu_1} \quad (7.3.3.2)$$

$\kappa$  = viscosity ratio  
 $\nu_1$  = required viscosity  
 $\nu$  = actual viscosity

### 7.3.4 Contamination & fatigue load limit

The contamination degree is tricky to estimate. The guideline from SKF is shown in Table (7.3.4). The pre-greased bearings in Flygt's products with their well protected surroundings have a contamination factor of 0.7. If not pre-greased, this value is 0.5.

Condition	$\eta_c$
<b>Very clean</b> Debris size of the order of the lubricant thickness	1
<b>Clean</b> Bearings greased for life and sealed	0.8 - 0.9
<b>Normal</b> Greased for life and shielded	0.5 - 0.8
<b>Contaminated</b> Bearing without seals; particles from surroundings	0.5 - 0.1
<b>Heavily contaminated</b> Intruding fluids and particles, extreme contamination	0

Table 7.3.4. Contamination factor  $\eta_c$

When loads, lubricant and environmental considerations have been calculated, there is time to get the adjustment factor to obtain the calculated lifetime of a bearing according to Equation (7.3.4).

$$\begin{aligned} L_{10mh} &= a_m \cdot L_{10h} \\ a_m &= f\left(\kappa, \eta_c \cdot \frac{P_u}{P}\right) \end{aligned} \quad (7.3.4)$$

$L_{10mh}$  = lifetime in hours  
 $L_{10h}$  = nominal lifetime in hours  
 $a_m$  = adjustment factor  
 $\kappa$  = viscosity ratio  
 $\eta_c$  = contamination factor  
 $P$  = equivalent dynamic load  
 $P_u$  = fatigue load limit

The adjustment factor is achieved from diagrams, such as the one shown in Figure (7.3.4.1), for different bearing types. The fatigue load limit is provided by the bearing manufacturer.

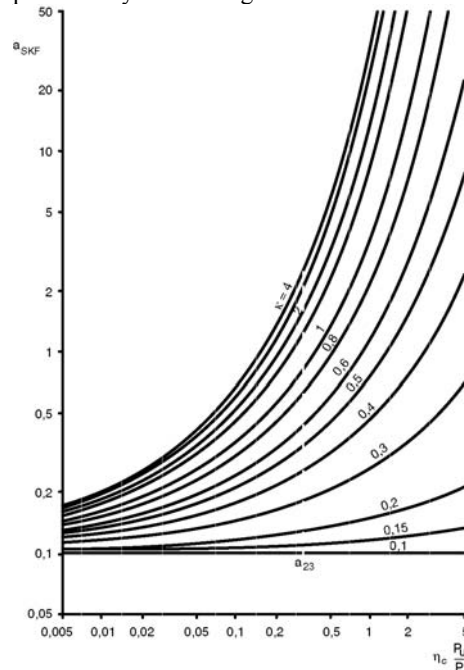


Figure 7.3.4.1. Adjustment factor  $a_m$  from SKF. Variables described in Equation (7.3.4)

The calculation can be made by hand with the help of diagrams, but it is much easier to use the online program available, shown in Figure (7.3.4.2).

**SKF** Print

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**Bearing life**

Select  $\eta_c$  0.7

Bearing **3308 A-2ZTN9**

$d$ , mm 40

$D$ , mm 90

$C$ , N 60500

$P_{01}$ , N 1830

$P_r$ , N 4980

$n$ , r/min 1750

$v$ , mm<sup>2</sup>/s 20

$K$	1.5	$L_{10}$	1793	$L_{10h}$	17076
$V_1$	13.3				
$a_{23}$	1.31	$L_{10a}$	2342	$L_{10ah}$	22306
$a_{SKF}$	18.9	$L_{10aa}$	33902	$L_{10aah}$	322876

Figure 7.3.4.2. An online calculation from SKF.

## 7.4 Lubrication life

The preceding information naturally depends on proper lubrication which itself has a specific lifetime. The lifetime of grease lubrication may be far shorter than the lifetime of a bearing. Some rules together with the diagram in Figure (7.4) (SKF) provide a rough estimation of the lifetime of the lubrication. The rules in short ( $t_f$  is lifetime in operating hours):

- ◆ If 90% reliability instead of 99%:  $t_f = t_f \cdot 2$
- ◆ For vertical mounting:  $t_f = t_f / 2$
- ◆ For every 15°C above 70°C:  $t_f = t_f / 2$
- ◆ For lower temperatures maximum:  $t_f = t_f \cdot 2$
- ◆ For synthetic grease above 70°C:  $t_f = t_f \cdot 4$
- ◆ For synthetic grease below 70°C:  $t_f = t_f \cdot 3$

Environment: If water or dirt can penetrate the bearing, the predicted lifetime should be lowered dramatically. If, on the other hand, the environment is extremely clean and no exchange with the outer world takes place, the lifetime can be prolonged ( $t_f = t_f \cdot 2$ ).

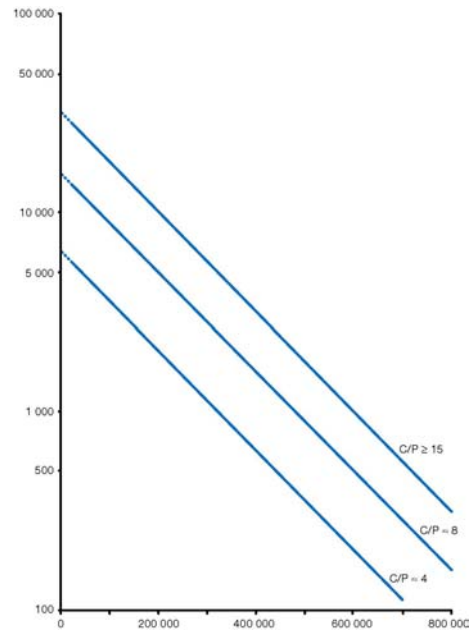


Figure 7.4. Time to relubrication (SKF) The y-axis is operating hours and the x-axis is the product of speed [RPM], bearing mean diameter [mm] and a factor 1 for normal ball bearings and 1,5 for normal roller bearings without axial load.

## 7.5 Practical life

The calculated lifetime, although correctly done, is just one part in estimating the practical life of a bearing. Moderately loaded bearings may work for a long time even though signs of fatigue have appeared. Matters like wear and all the causes of failure mentioned in Section 2.3 shall also be considered. A calculated lifetime that is too long can even shorten the practical life of a bearing. The goal for a Flygt bearing to meet is:

**ITT Flygt's bearings shall guarantee a service interval of 50 000 hours and be seen "as trouble free as a bolt".**

In product development, we may also use x-ray diffraction to analyze bearings exposed to real loads in a field test. The analysis gives information

of loading and remaining lifetime and thereby enhances lifetime estimations.

### 7.5.1 Wear

Particles do not only create high stresses in the bearing and lower the calculated lifetime, they may also cause wear, another reason for cleanliness of bearings and their lubrication. The wear rate depends mainly on the particle concentration, particle size, particle hardness and sliding.

### 7.5.2 Lubrication

Lubrication properties are of great importance and Flygt continuously tries to get the best lubrication for our products. One should note that grease is perishable and one test Flygt performs to compare different grease in this respect is a heat test at 140°C. This test and others, as well as tests from lubricant and bearing suppliers, give the basis from which we predict lubricant lifetime.

## 8. Safety factors

To determine the appropriate safety factor to use, there are many considerations to take into account; type of failure, accuracy in calculations, acceptance level of failure, etc. This can be structured as a set of factors that can be multiplied to form a safety factor as shown below:

- ◆ Demands 1. Factor for failure probability is 1 if less serious (failure probability of  $10^{-3}$ ), 1.1 if serious (failure probability of  $10^{-4}$ ) and 1.2 if very serious (failure probability of  $10^{-5}$ ).
- ◆ Demands 2. Factor for control of calculation, material, manufacturing and use is 1 if rigorous control, 1.05 if normal, and 1.1 if low degree of control.
- ◆ Load. Factor for load is 1 if maximum values, loads with limitations, or fixed load with no variations are used. For other conditions, an estimation of the load probability has to be made.
- ◆ Resistance 1. Factor for material is 1 if guaranteed minimum values or probability of lower values are less than  $10^{-3}$ . 1.1 to 1.3 if normal distribution and average values of ultimate stress, yield stress and crack propagation values. 1.3 to 1.7 if normal distribution and average values of fatigue data for infinite life.
- ◆ Resistance 2. Factor for calculation method is 1 if worst case calculation. 1.1 to 1.2 if handbook formulas or FEA. It is larger than 1.2 if less accurate methods are used.
- ◆ Resistance 3. Factor for criteria is 1 if plastic deformation or non-brittle failure, if

brittle failures and one-directional stress, if fatigue and infinite life or if fatigue and constant amplitude at one-directional stress.

1.1 if brittle failure and multi-directional stress, if fatigue and limited life or if fatigue and multi-directional stress. 2 if fatigue and several different load cases. Time independent deformation gives a factor of 1, time dependent deformations 1.1 to 2.

The figures above are given as guidance. If the probability distribution of the factors above is known then a more sophisticated analysis may be performed. This is normally not the case, however. The final safety factor is formed by multiplying the two demand factors, the load factor and the three resistance factors.

If a component is tested, like bearings, then the values obtained from tests are the ones to use. This type of endurance test is normally not possible to perform, especially not for components produced in a low number.

## 9. Conclusions

When analyzing a rotating system, there are many calculations involved and assumptions to be made. This publication shows how Flygt performs this and what methods Flygt considers to be the most suitable. In order to compare predictions of bearing life, safety levels of different failures, etc., it is necessary to know how they have been derived.

The amount of data needed (and generated) to obtain results from one duty point of one combination of drive unit and hydraulic part is huge. For the entire pump curve, even more is needed. Consequently, it is impossible to analyze all combinations of all impellers/propellers, volutes, drive units and running conditions.

What finally counts is the performance of the product, not a good figure of some kind. Especially since there are a lot of failure conditions that are unpredictable by calculations. Nevertheless, accurate analyses are necessary to compare different designs and to create a robust product that fulfills the customer's demands.

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